Description of readout processes during strong beam coupling

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We show, using the symmetry properties of the coupled-wave equations for the transmission and reflection geometries, that any readout characteristic of dynamic spatially nonuniform index gratings in photorefractive crystals can be explicitly expressed through the characteristics of the recording light beams. This approach is applied to describe the impact of beam coupling on the diffraction efficiency of dynamic gratings and on the output intensities of the light beams at instantaneous input phase changes (light grating translation). Further implications of this general approach are discussed.

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I. INTRODUCTION

Photorefractive nonlinear-optical phenomena are caused by dynamic processes of buildup of refractive index gratings and Bragg diffraction of light waves from these gratings [1,2]. They include wave amplification, phase conjugation, light scattering, optical oscillations, and many other important effects. The major constituents of grating formation are charge separation under light and the linear electro-optic effect. The relevant materials include numerous photosensitive crystals and polymers [2–4].

Typically, the photorefractive nonlinearity is already strong under the conditions of CW experiments. At the same time, it is rather inertial owing to the slowness of charge separation under low light intensities [2,3]. This distinctive feature allows to employ effective methods for the testing and control of nonlinear effects. Imagine that the input conditions for the incident light beams are changed during a time period which is much shorter than the characteristic response time of medium. The refractive index profile remains practically unchanged and the changes of the output optical characteristics are fully due to different ways of probing the recorded spatial grating.

The simplest testing method consists in measuring the diffraction efficiency of the dynamic grating by instantaneous blocking one of the incident recording light beams [2]. This instantaneous diffraction efficiency contains information on the amplitude, but not the spatial position (the phase) of the grating. The so-called grating translation technique allows to measure *in situ* both the amplitude and phase of dynamic index gratings [4–6]. This method employs a strong momentary phase modulation of one of the input beams which changes the position of the light fringes. The third important implication of the readout properties is the feedback-controlled beam coupling [7–10]. In this case, the readout characteristics obtained with the help of an auxiliary weak phase modulation are used to adjust the input phase via an electronic feedback loop. This loop stabilizes photorefractive setups and, at the same time, modifies strongly the character of beam coupling.

Until recently, employment of the readout characteristics was overwhelmingly based on the Kogelnik relations for spatially uniform index gratings [11]. In other words, the dynamic distortions of the grating amplitude and phase were neglected. This approach is justified for sufficiently thin samples exhibiting weak nonlinear-optical effects but it is not valid when the energy and/or phase exchange between the recording beams is strong. These cases are indeed of great importance for applications.

To find the readout characteristics in the simplest case of two-wave coupling, see Fig. 1, it is necessary, in the general case, to calculate the grating profile at the readout moment and to solve the coupled-wave equations for the light amplitudes once more using the boundary conditions relevant to the particular readout process. A few particular solutions to this readout problem are known to date. Steady state diffraction efficiency has been calculated for the transmission geometry using a particular microscopic model of the photorefractive response [12]. It was found that the nonlinear distortions lead to strong modifications of the Kogelnik relations. The importance of taking into account the nonlinear distortions for analysis of the grating translation technique data was recognized recently [13]. Controversial attempts to take these corrections into account are presented in Ref. [14].

The first aim of this paper is to show that the symmetry of the conventional coupled-wave equations for two-wave mixing enables one to express algebraically any readout characteristic through the output characteristics of the recording light waves. In other words, the readout problem is reduced to the problem of wave-coupling modeling. Correspondingly, the tedious procedure of resolving the coupled-wave equations with nonconstant coefficients becomes unnecessary. This is valid for both transmission and reflection geometries of wave coupling. Furthermore, the general relations obtained at this stage are sufficient to formulate properly the problem of feedback controlled beam coupling.

The second aim is to apply the general relations to particular cases to illustrate the influence of beam coupling on the diffraction efficiency and the characteristics of the grating translation technique. Discussion of experimental details and fitting experimental curves is beyond the scope of this paper.

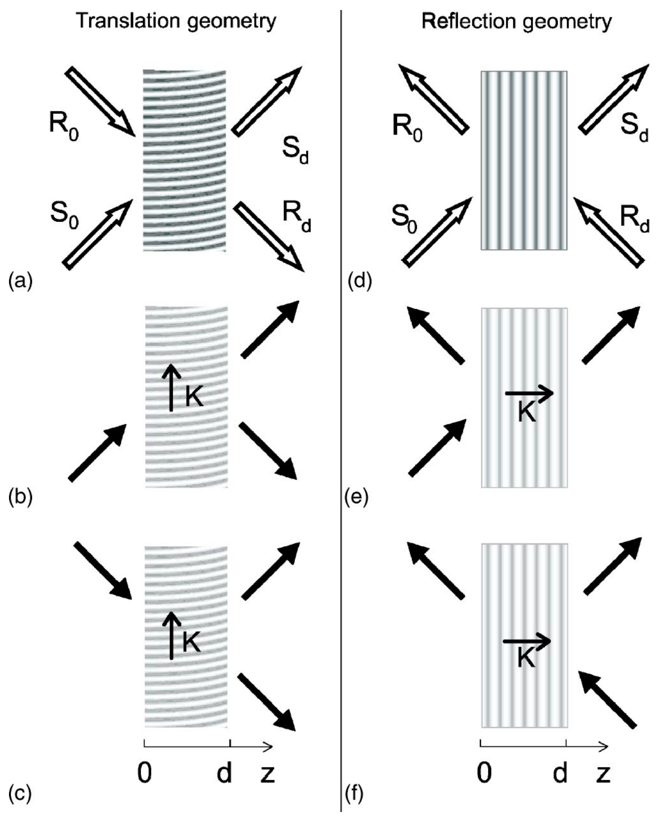

FIG. 1. Schematic representation of the main processes for the *T* geometry (a–c) and the *R* geometry (d–e). The subfigures (a) and (d) depict the recording processes, whereas the subfigures (b), (c), (e), and (f) illustrate the readout processes relevant to the fundamental amplitudes $\tilde{S}_{s,r}(z)$ and $\tilde{R}_{s,r}(z)$. The parallel lines show the grating fringes.

The symmetry properties of the coupled-wave equations have been used recently for analysis of particular photorefractive effects [15,16]. A full-scale formulation of the novel approach to the description of the readout processes is, however, still missing. Most part of the results presented below are different; a few known particular results are reproduced with the different method to exhibit its efficiency.

II. BASIC RELATIONS

We suppose that two light waves of the same frequency, referred to as signal and reference waves, propagate in a photorefractive nonlinear medium. Their amplitudes, **S** and **R** are slowly varying functions of the time t and the propagation coordinate z. The waves are coupled via Bragg diffraction from the electro-optic grating $E_{sc}(\vec{r})$,

$$E_{sc} = E_K\, e^{i\vec{K}\cdot\vec{r}} + \text{c.c.} \tag{1}$$

The grating vector $\vec{K}$ is the difference of the light wave vectors, and the grating amplitude E_K depends generally on z and t.

In what follows we consider the transmission (T) and reflection (R) coupling geometries, see Figs. 1(a) and 1(d). In the T and R cases the grating vector $\vec{K}$ is perpendicular and parallel to the z axis, whereas the z components of the light wave vectors are of the same and of the opposite signs, respectively.

For the T geometry the coupled-wave equations for the light amplitudes **S** and **R** can be presented in the form [2]

$$d\mathbf{S}/dz = i\kappa E_K \mathbf{R},$$

$$d\mathbf{R}/dz = i\kappa E_K^* \mathbf{S}, \tag{2}$$

where $\kappa = \pi n^3 r/\lambda$, n is the nonperturbed refractive index, λ is the light wavelength, r is the relevant electro-optic coefficient, and the asterisk means taking the complex conjugate. The overall intensity I_0 is constant during propagation in the T case, $I_0 = |\mathbf{S}(z)|^2 + |\mathbf{R}(z)|^2 = \text{const}$. It can be regarded as the input intensity.

For the R geometry the coupled-wave equations are

$$d\mathbf{S}/dz = i\kappa E_K \mathbf{R},$$

$$d\mathbf{R}/dz = -i\kappa E_K^* \mathbf{S}. \tag{3}$$

The minus sign in the second line stems from the negative propagation direction of the R wave. The difference of the light intensities Δ is conserving during propagation in this case, $\Delta = |\mathbf{R}(z)|^2 - |\mathbf{S}(z)|^2 = \text{const}$.

The fact that the sets (2) and (3) do not include explicitly the time variable t means that light follows adiabatically slow changes of the grating amplitude $E_K = E_K(z,t)$.

The coupled-wave equations can be applied to two different physical problems.

The first one is determination of the readout characteristics of the grating. The grating amplitude $E_K(z,t)$ is treated here as a known complex function, and the differential equations for **S** and **R** are to be solved with proper boundary conditions. While the readout problem is linear, it cannot be solved analytically (or in quadratures) in the general case. The well known particular case when Eqs. (2) or/and (3) can be solved is the case of a spatially uniform grating, $E_K(z) = \text{const}$. The corresponding simple relations for **R** and **S** are known as Kogelnik theory [11]. They are often in use even in the situations where the assumption of spatial uniformity cannot be justified.

The second problem is the description of beam coupling during grating recording. In this case, the set (2) or (3) has to be supplemented by a material equation, which couples the grating amplitude E_K with the light amplitudes **R** and **S**. The particular form of this model equation, which depends on the charge transport properties of the material, is not important for us at this stage. It is essential, however, that the characteristic response (buildup) time entering the material equation is long enough to allow short-time changes of the boundary conditions for the light amplitudes (changes of the readout conditions) without significant changes in E_K. The characteristic time ranges typically from 10^{-2} to 10^{2} s in photorefractive continuous-wave experiments [1,2]. Another important feature is that the grating does not remain spatially uniform when the beam coupling is strong.

Our main concern is the readout characteristics of the grating during strong beam coupling. The standard approach to this problem is as follows: First, using analytical or/and numerical tools, it is necessary to solve the nonlinear problem of beam coupling to find the light amplitudes $\mathbf{R}$ and $\mathbf{S}$. Second, using again the material equation, to find the dependence $E_K(z)$. Third, to substitute E_K into the readout equations and solve them once more with new boundary conditions for $\mathbf{R}$ and $\mathbf{S}$.

We will show below that knowledge of the amplitudes of the recording waves (or even combinations of these amplitudes) allows to solve explicitly any readout problem using pure algebraic means.

III. READOUT CHARACTERISTICS

Until this point our notation $\mathbf{S}$, $\mathbf{R}$ for the light amplitudes was general. From now on we specialize it to avoid confusion. We shall set $\mathbf{S}=S$, $\mathbf{R}=R$ for the amplitudes of the recording waves and $\mathbf{S}=\tilde{S}$, $\mathbf{R}=\tilde{R}$ for the light amplitudes during readout. Furthermore, we shall consider sequentially the T and R cases.

A. General properties: Transmission geometry

Let R and S be the amplitudes of the reference and signal waves during recording and E_K be the corresponding grating amplitude. These quantities are generally functions of z and t. It is implied thus that we know a particular solution of the set (2) for R and S which corresponds to a particular choice of the input light amplitudes and to a particular (nonspecified) material equation.

To describe the readout characteristics of the grating at an arbitrary time moment t, we have to solve the set (2) for $\mathbf{S}=\tilde{S}$ and $\mathbf{R}=\tilde{R}$ with the same spatial profile $E_K(z)$ but new input values of the light amplitudes. The general solution of this linear problem can, as known from basic mathematics, be presented as a linear combination of two particular independent solutions for $\tilde{R}$ and $\tilde{S}$. One particular solution is known, this is the solution for the recording amplitudes, $\tilde{R}_{part1}=R$, $\tilde{S}_{part1}=S$.

To find the second particular solution, we make the complex conjugation of Eqs. (2). One can make sure then that the pair $\tilde{R}_{part2}=S^*$, $\tilde{S}_{part2}=-R^*$ represents this solution for the T geometry. The above relations stem indeed from the symmetry properties of the coupled-wave equations.

Consequently, the general solution of the readout problem is

$$\begin{pmatrix}\tilde{S}\\ \tilde{R}\end{pmatrix} = c_1\begin{pmatrix}S\\ R\end{pmatrix} + c_2\begin{pmatrix}-R^*\\ S^*\end{pmatrix}, \tag{4}$$

where c_1 and c_2 are arbitrary constants. As soon as the boundary conditions for a particular readout process [for $\tilde{S}(0)$ and $\tilde{R}(0)$] are formulated, one can express algebraically $c_{1,2}$ through the input and output values of the recording amplitudes and calculate in the next step the output values $\tilde{S}(d)$ and $\tilde{R}(d)$, where d is the thickness of the sample, see Fig. 1(a).

One kind of readout problem plays a fundamental role for understanding of the properties of diffraction and transmission through thick dynamic index gratings and also for the description of many other readout processes. Let the incident R beam be blocked and the grating recorded to the moment t be tested by the S beam of a unit amplitude, see Fig. 1(b). We denote as the corresponding fundamental amplitudes $\tilde{S}_s$ and $\tilde{R}_s$; they are functions of the propagation coordinate z and time t. The boundary conditions for them are $\tilde{S}_s(0)=1$, $\tilde{R}_s(0)=0$. The convenience of our notation for the fundamental amplitudes becomes clear if the reader accepts a simple mnemonic rule — the subscript marks the type of the only readout beam. Using Eq. (4), we obtain $c_1=S_0^*/I_0$, $c_2=-R_0/I_0$ and therefore

$$\tilde{S}_s = (S_0^*\, S + R_0\, R^*)/I_0,$$

$$\tilde{R}_s = (S_0^*\, R - R_0\, S^*)/I_0, \tag{5}$$

where $S_0=S(z=0)$ and $R_0=R(z=0)$.

Similarly we introduce the fundamental amplitudes $\tilde{S}_r$, $\tilde{R}_r$, that correspond to testing of the same grating by the R beam of a unit amplitude and meet the boundary conditions $\tilde{S}_r(0)=0$, $\tilde{R}_r(0)=1$, see Fig. 1(c). One can make sure that

$$\tilde{S}_s = \tilde{R}_r^*, \quad \tilde{S}_r = -\tilde{R}_s^*. \tag{6}$$

One pair of the fundamental amplitudes is easily expressed through the other. It is evident also that

$$|\tilde{S}_s|^2 + |\tilde{R}_s|^2 = |\tilde{S}_r|^2 + |\tilde{R}_r|^2 = 1. \tag{7}$$

The fundamental amplitudes fully characterize the diffraction properties of the grating in an absolute scale. At the same time, there is one-to-one correspondence between them and the recording amplitudes. And lastly, the fundamental amplitudes provide a highly useful basis for decomposition of the light amplitudes (during recording or readout) into the transmitted and diffracted components. This important issue becomes evident when we express the amplitudes $\mathbf{R}$ and $\mathbf{S}$ through $\tilde{S}_{s,r}$ and $\tilde{R}_{r,s}$,

$$\mathbf{S}(z) = \mathbf{S}_0\, \tilde{S}_s(z) + \mathbf{R}_0\, \tilde{S}_r(z),$$

$$\mathbf{R}(z) = \mathbf{R}_0\, \tilde{R}_r(z) + \mathbf{S}_0\, \tilde{R}_s(z). \tag{8}$$

In accordance with the definition of the fundamental amplitudes, see also Figs. 1(b) and 1(c) $\mathbf{S}_0\, \tilde{S}_s(z)$ and $\mathbf{R}_0\, \tilde{S}_r(z)$ are the transmitted and diffracted components of the S beam while $\mathbf{R}_0\, \tilde{R}_r(z)$ and $\mathbf{S}_0\, \tilde{R}_s(z)$ are the transmitted and diffracted components of the R beam.

The diffraction efficiency of the refractive index grating η is the simplest experimental readout characteristic. Let one of the incident writing beams, see Fig. 1(a), be blocked for a moment. Then η is defined as the intensity ratio of the output diffracted beam to the single input beam. In accordance with this definition we have

$$\eta = |\tilde{R}_s(d)|^2 = |\tilde{S}_r(d)|^2. \quad (9)$$

The above relations prove also that irrespective of the spatial profile $E_K(z)$ the result of measurement of η does not depend on which of the input beams is blocked.

Using Eqs. (5) and (6) we represent the diffraction efficiency in the following explicit form:

$$\eta = \frac{\beta_0 + \beta_d - 2(\beta_0\beta_d)^{1/2} \cos \psi}{(1+\beta_0)(1+\beta_d)}, \quad (10)$$

where $\beta_0 = |R_0|^2/|S_0|^2$ and $\beta_d = |R_d|^2/|S_d|^2$ are the input and output intensity ratios for the recording beams while $\psi = \arg(R_d S_d R_0^* S_0^*) = \varphi_d^r + \varphi_d^s - \varphi_0^r - \varphi_0^s$ is a combination of their input and output phases.

Consider now the grating translation technique. It consists of introduction of a variable momentary phase shift φ into one of the input writing beams. Let it be the signal beam. Then the boundary conditions during readout are $\tilde{S}_0 = S_0\, e^{i\varphi}$, $\tilde{R}_0 = R_0$. Using Eqs. (8) we obtain immediately the general relations for the output amplitudes:

$$\tilde{S}(d) = S_0\, e^{i\varphi}\, \tilde{S}_s(d) + R_0\, \tilde{S}_r(d),$$

$$\tilde{R}(d) = R_0\, \tilde{R}_r(d) + S_0\, e^{i\varphi}\, \tilde{R}_s(d). \quad (11)$$

In an experiment the measurable quantities are the changes of the output intensities at the phase scanning. The output intensity changes $\delta I_S(\varphi) = [|\tilde{S}(d)|^2 - |S(d)|^2]$, $\delta I_R(\varphi) = [|\tilde{R}(d)|^2 - |R(d)|^2]$ can be presented in the form

$$\delta I_{S,R}/I_0 = \pm [A^T \sin \varphi + B^T(1 - \cos \varphi)], \quad (12)$$

which is consistent with the conservation of the total output intensity. The dimensionless parameters A^T and B^T characterize the output intensity oscillations which are symmetric and asymmetric against the zero level, respectively; these coefficients can be determined experimentally. Using Eqs. (5) and (11) we express them explicitly through the recording characteristics,

$$A^T = (m_0 m_d/2)\sin \psi,$$

$$B^T = (m_0\, w_d - m_d\, w_0 \cos \psi)/2, \quad (13)$$

where $m = 2|SR^*|/I_0$ is the contrast of the recording light interference pattern, $w = (|R|^2 - |S|^2)/I_0$ the normalized intensity difference for the recording beams, and the subscripts 0 and d mean, as usual, taking the recording characteristics at $z = 0$ and d. The phase difference ψ has been introduced earlier. The parameters m, w, and β are coupled with each other by the simple relations: $m = (1 - w^2)^{1/2} = 2\beta^{1/2}/(\beta+1)$, $w = (\beta - 1)/(\beta+1)$; the use of these parameters is the matter of convenience.

One more application of our approach is the description of the feedback-controlled beam coupling. In this case an electronic feedback loop adjusts the input phase of the recording S beam, φ_s^0, in such a way to keep the phase difference Φ between the diffracted and transmitted components of the output signal beam equal $\pi/2$ or $-\pi/2$. Experimental implementation of this idea was correct from the beginning of the studies [7] but an adequate description of the operation mode was accomplished only recently [10]. It is based on the notion of the fundamental amplitudes.

To explain the operation principle, we make use of Eqs. (8) to decompose the recording amplitude $\mathbf{S} = S$ into the sum of the transmitted and diffracted components. According to this presentation, the output phase difference Φ between these components is

$$\Phi(t) = \varphi_r(0,t) - \varphi_s(0,t) + \arg[\tilde{S}_r(d,t)\tilde{S}_s^*(d,t)]. \quad (14)$$

The condition $\Phi = \pm\pi/2$ can be satisfied by adjustment of $\varphi_s^0(t)$ [with $\varphi_r^0(t) = \text{const}$] unless the product $|\tilde{S}_r(d)\tilde{S}_s^*(d)| = \sqrt{\eta(1-\eta)}$ is not zero. In the case of $\eta = 0$ or 1 the phase difference Φ makes no sense and the phase adjustment is not possible.

Our approach allows thus to formulate the feedback problem irrespective of the nonlinear distortions of the refractive index profile. It is the true basis for describing of the feedback controlled beam coupling. The results of nonlinear modeling are in accordance with the experimental observations; they reveal highly unusual features of the feedback controlled beam coupling [10]. The formulation of the feedback conditions in the terms of spatially uniform index grating, used initially, is not valid for the samples providing high values of the diffraction efficiency.

How to implement the feedback condition $\Phi = \pm\pi/2$ experimentally? An auxiliary oscillating component $\delta\varphi_s^0 = q \cos \omega t$ has been introduced into the input phase of the signal wave to accomplish the feedback [7,9]. The oscillation amplitude q is very small and the modulation frequency ω exceeds considerably the inverse response time of the medium. The effect of this auxiliary phase shift on the recording process is negligible but it is sufficient for readout. In accordance with Eqs. (8), the output intensity of the signal wave acquires the contributions oscillating as $\sin \omega t$ and $\cos 2\omega t$. The amplitudes of the intensity oscillations are $I_\omega = 2|S_0 R_0|\sqrt{\eta(1-\eta)}q \sin \Phi$ and $I_{2\omega} = 0.5|S_0 R_0|\sqrt{\eta(1-\eta)}\, q^2 \cos \Phi$. Using $I_{2\omega}$ as an error signal and controlling the sign of I_ω, one can keep electronically the phase difference Φ equal to $\pi/2$ or $-\pi/2$.

B. General properties: Reflection geometry

With some changes, the results of the preceding section can be employed for the R case.

If we consider the pair of the recording amplitudes R, S as the first basic vector for constructing the general solution for $\tilde{R}$ and $\tilde{S}$, the second basic vector can be chosen as S^*, R^*. It is evident from the structure of the complex conjugated set (3). Consequently, the general solution to the readout problem is

$$\begin{pmatrix} \tilde{S} \\ \tilde{R} \end{pmatrix} = c_1 \begin{pmatrix} S \\ R \end{pmatrix} + c_2 \begin{pmatrix} R^* \\ S^* \end{pmatrix}, \qquad (15)$$

compare with Eqs. (4).

In the next step we find out the fundamental amplitudes characterizing the diffraction and transmission properties of the reflection grating at an arbitrary time moment t. The fundamental amplitudes $\tilde{S}_s(z)$, $\tilde{R}_s(z)$ meet the boundary conditions $\tilde{S}_s(0)=1$, $\tilde{R}_s(d)=0$, they correspond to readout of the grating with the S beam of a unit amplitude, see Fig. 1(e). The second pair of the fundamental amplitudes, $\tilde{S}_r(z)$, $\tilde{R}_r(z)$, meets the boundary conditions $\tilde{S}_r(0)=0$, $\tilde{R}_r(d)=1$; it corresponds to readout of the same grating with the R beam of a unit amplitude, Fig. 1(f). Since the difference of the light intensities is conserving in the R case, we have

$$|\tilde{S}_s|^2 - |\tilde{R}_s|^2 = |\tilde{R}_r|^2 - |\tilde{S}_r|^2, \qquad (16)$$

compare with Eq. (7).

Using Eq. (15) one can express algebraically the fundamental amplitudes through the recording amplitudes R and S. We have

$$\tilde{S}_s = (S_d^* S - R_d R^*)/I_1,$$

$$\tilde{R}_s = (S_d^* R - R_d S^*)/I_1, \qquad (17)$$

for $\tilde{S}_s$ and $\tilde{R}_s$, and

$$\tilde{S}_r = (S_0 R^* - R_0^* S)/I_1,$$

$$\tilde{R}_r = (S_0 S^* - R_0^* R)/I_1, \qquad (18)$$

for $\tilde{S}_r$ and $\tilde{R}_r$, where $I_1 = S_0 S_d^* - R_0^* R_d$ is a complex constant.

As follows from here, the pairs $\tilde{S}_s, \tilde{R}_s$ and $\tilde{S}_r, \tilde{R}_r$ are coupled with each other by the relations $\tilde{S}_s = \tilde{R}_r^*(0 \leftrightarrow d)$, $\tilde{S}_r = \tilde{R}_s^*(0 \leftrightarrow d)$, which are similar to the relations (6) for the T case. The sign $\leftrightarrow$ means interchanging of the subscripts 0 and d. Representation of the amplitudes as the sums of the diffracted and transmitted components, which is similar to that given by Eqs. (8), also holds true:

$$\mathbf{S}(z) = \mathbf{S}_0 \, \tilde{S}_s(z) + \mathbf{R}_d \, \tilde{S}_r(z),$$

$$\mathbf{R}(z) = \mathbf{R}_d \, \tilde{R}_r(z) + \mathbf{S}_0 \, \tilde{R}_s(z). \qquad (19)$$

It is applicable to both recording and readout processes. Its meaning is illustrated by Figs. 1(d)–1(f).

The diffraction efficiency of the reflection grating is given by

$$\eta = |\tilde{R}_s(0)|^2 = |\tilde{S}_r(d)|^2. \qquad (20)$$

As follows from here, the value of η does not depend on which of the input writing beams (S or R) is blocked.

Using Eqs. (17) and (18) we express η by the intensity ratio for the recording waves $\beta = |R|^2/|S|^2$ (taken at $z=0$ and d) and the phase difference $\psi = \arg(R_d S_d R_0^* S_0^*)$,

$$\eta = \frac{(\beta_d/\beta_0)^{1/2} + (\beta_d/\beta_0)^{-1/2} - 2\cos\psi}{(\beta_d\beta_0)^{1/2} + (\beta_d\beta_0)^{-1/2} - 2\cos\psi}. \qquad (21)$$

The general expression for the output amplitudes, which is relevant to the phase modulation of the input S beam during readout (the grating translation technique) follows from Eqs. (19):

$$\tilde{S}(z) = S_0 \, e^{i\varphi} \, \tilde{S}_s(z) + R_d \, \tilde{S}_r(z),$$

$$\tilde{R}(z) = R_d \, \tilde{R}_r(z) + S_0 \, e^{i\varphi} \, \tilde{R}_s(z). \qquad (22)$$

The output intensity changes $\delta I_S(\varphi) = [|\tilde{S}(d)|^2 - |S(d)|^2]$ and $\delta I_R(\varphi) = [|\tilde{R}(0)|^2 - |R(0)|^2]$ can be presented in the form

$$\delta I_{S,R}/I_{in} = \pm [A^R \sin\varphi + B^R(1 - \cos\varphi)], \qquad (23)$$

where $I_{in} = |S_0|^2 + |R_d|^2$ is the total input intensity for the recording waves. Using Eqs. (17) and (18) and recalling that the intensity difference is the conserving quantity, we obtain for the modulation amplitudes A^R and B^R:

$$A^R = \frac{g \sin\psi}{(\beta_0\beta_d)^{1/2} + (\beta_0\beta_d)^{-1/2} - 2\cos\psi},$$

$$B^R = \frac{g[(\beta_d/\beta_0)^{1/2} - \cos\psi]}{(\beta_0\beta_d)^{1/2} + (\beta_0\beta_d)^{-1/2} - 2\cos\psi}, \qquad (24)$$

with $g = 2(1-\beta_0)(1-\beta_d)/(1-\beta_0\beta_d)$. Again, the output intensity changes during the translation are expressed explicitly through the input and output intensity ratios during recording and the phase ψ.

Formulation of the feedback conditions for the R geometry is not much different from this described above for the T case. The first results on the feedback controlled beam coupling in the reflection geometry have been reported only recently [17,18].

It is worth mentioning that all above relations are free of model assumptions about the recording process. In other words, they are applicable to any particular model of the grating formation. As soon as this model is specified and the recording characteristics are calculated (in steady state or during a transient process), we can describe immediately all readout characteristics. Below we consider representative examples of material models.

IV. PARTICULAR RESULTS

A. Models of nonlinear response

By applying the general relations, we shall restrict ourselves to the case of steady state. We shall assume the following fairly general relation between the grating amplitude E_K and the recording light amplitudes S and R [1,2],

$$E_K = \gamma \frac{SR^*}{|S|^2 + |R|^2}, \tag{25}$$

where $\gamma = \gamma' + i\gamma'' = |\gamma| \exp(i\theta)$ is a complex parameter characterizing the type and strength of the photorefractive nonlinear response. The absolute value of the ratio in the right-hand side is the half contrast of the recording light interference pattern, $m/2 = |SR^*|/(|S|^2 + |R|^2)$. The value of $\theta = \arg(\gamma)$ is nothing else than the phase shift between the light and grating fringes. If γ is real, the phase shift θ is 0 or π; this is the case of local nonlinear response. If γ is imaginary, the phase shift is $\pm\pi/2$; this corresponds to the nonlocal response. In the general case γ is complex, i.e., the nonlinear response is mixed.

It is possible to express γ through the applied electric field (if present), the grating vector K, and the material parameters such as mobility-lifetime product for photoexcited carriers and the trap concentration using particular microscopic models of light-induced charge transport [1–3]. Determination of the dependences of γ', γ'' (or $|\gamma|, \theta$) on the variable experimental parameters allows one to judge about the mechanisms of charge transfer.

Calculation of the necessary steady-state recording characteristics does not present serious difficulties. Taking into account the conservation laws for the T and R cases, it is possible to find at first the light intensities and to calculate then the phase dependence $\psi(z)$.

To get a reference point for analysis of the role of coupling effects, we shall consider also a model which ignores the influence of beam coupling on the grating amplitude. Within this uniform-grating model the grating amplitude is constant, $E_K = E_K^0$, where

$$E_K^0 = \gamma \frac{S_{in} R_{in}^*}{|S_{in}|^2 + |R_{in}|^2}, \tag{26}$$

The subscript *in* means taking the input values of the recording amplitudes. For the T geometry we have $S_{in} = S_0$, $R_{in} = R_0$, whereas in the R case one should set $S_{in} = S_0$, $R_{in} = R_d$.

Employment of the general relations for the readout characteristics within the uniform-grating model gives no real advantages because of the simplicity of the direct calculations of the amplitudes $\tilde{R}(z)$ and $\tilde{R}(z)$. It is useful merely to make sure that the general relations lead here to the known results of the Kogelnik theory [11].

As follows from the structure of Eqs. (25) and (26) and the coupled-wave equations (2) and (3), the parameters γ and κ enter the output characteristics via the dimensionless product $p = \kappa\gamma d$. We shall refer to the complex parameter $p = p' + ip''$ as to the coupling strength. The absolute value of the coupling strength, $|p|$, can easily exceed 5–10 in photorefractive experiments [1,2].

B. Transmission geometry

Solution of Eqs. (2) and (25) for the recording amplitudes results in the following relations for the output intensity ratio $\beta_d = |R_d|^2/|S_d|^2$ and the phase $\psi = \arg(R_d S_d R_0^* S_0^*)$:

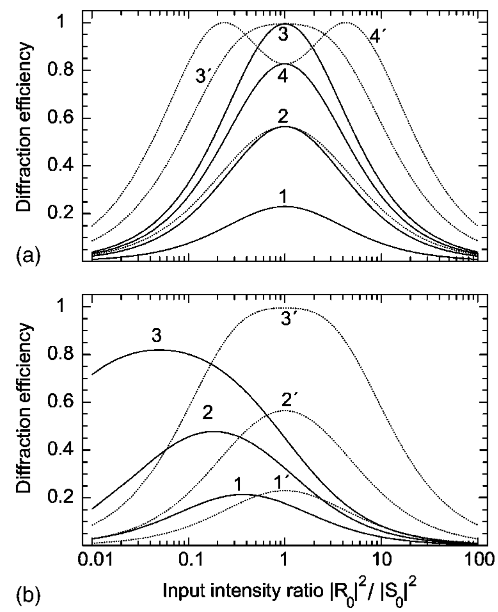

FIG. 2. Diffraction efficiency versus the input intensity ratio for the T geometry and the cases of local (a) and nonlocal (b) nonlocal response. The solid and dotted lines correspond to the exact relations and the uniform-grating model. The curves $1, 1'$, $2, 2'$, and $3, 3'$ are plotted for the values of the coupling strength 1, 1.7, and 3, respectively.

$$\beta_d = \beta_0 \, e^{2p''}, \quad \psi = p'. \tag{27}$$

The energy exchange between the recording beams is controlled by the imaginary part of the coupling strength p'' whereas the real part p' is responsible for the phase exchange.

By substituting Eqs. (27) into Eq. (10) we obtain for the diffraction efficiency,

$$\eta = \frac{m_0}{2} \frac{\cosh p'' - \cos p'}{\cosh(p'' + p_0)}, \tag{28}$$

where $m_0 = 2\sqrt{\beta_0}/(1+\beta_0)$ is the input light contrast and $p_0 = \ln\sqrt{\beta}$. This is not different from the result obtained by the direct calculations with no use of the symmetry properties [12].

Within the uniform-grating model, see Eq. (26), the diffraction efficiency is given by the Kogelnik expression, $\eta = \sin^2(m_0|p|/2)$. For nonlinearly thin samples, $|p| \equiv |\kappa\gamma d| \lesssim 1$, it gives the same result as Eq. (28), namely, $\eta \simeq (m_0|p|/2)^2$. For nonlinearly thick crystals the effects of beam coupling become important.

The solid lines in Fig. 2(a) show the function $\eta(\lg \beta_0)$ for the case of local response $(p''=0)$ and several values of $p' = \kappa\gamma' d$. This function is even and it does not depend on the sign of p'. The dotted lines show the dependences calculated within the uniform-grating model. One sees that the influence of the coupling effects is absent for $\beta_0 = 1$. When $\beta_0 \neq 1$ the coupling effects become essential for $|p'| \gtrsim 2$. In contrast to the Kogelnik theory, the exact relation predicts decrease of η with increasing $|\lg \beta_0|$ for any value of p'.

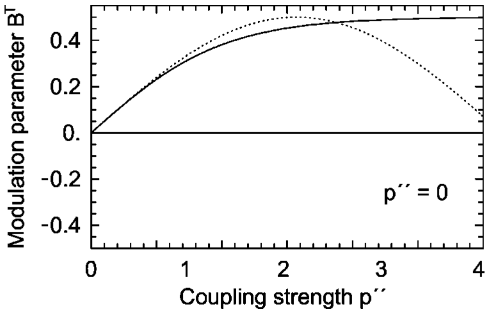

FIG. 3. Dependence of the modulation amplitude B^T on the coupling strength p'' for the nonlocal response in the T geometry; the dotted line is plotted for the uniform-grating model.

The influence of the coupling effects is even more pronounced in the case of nonlocal response, $p'=0$, see Fig. 2(b). The function $\eta(\lg \beta_0)$ is not even here because of the unidirectional energy transfer [1,2]; the diffraction efficiency can approach unity only for large values of $|p''|$. The replacement $p'' \to -p''$ results in the transformation $\eta(\lg \beta_0) \to \eta(-\lg \beta_0)$.

Now we turn to the grating translation technique. By substituting Eqs. (27) into Eqs. (13) we obtain for the modulation parameters A^T, B^T entering the general relation (12):

$$A^T = \frac{m_0 \sin p'}{2 \cosh(p''+p_0)},$$

$$B^T = \frac{m_0[\sinh(p''+p_0) - \sinh p_0 \cos p']}{2 \cosh(p''+p_0)}. \qquad (29)$$

Within the uniform-grating model these parameters are given by

$$A_0^T = m_0 p' \sin(m_0|p|)/2|p|,$$

$$B_0^T = m_0 p'' \sin(m_0|p|)/2|p|. \qquad (30)$$

For nonlinearly thin crystals, $|p| \lesssim 1$, Eqs. (29) and (30) give the same result.

Consider the case of equal input intensities, $m_0=\beta_0=1$, which is important for experiment. The expressions (29) become here especially simple, $A^T=\sin p'/2\cosh p''$, $B^T=\tanh p''/2$. They can serve as a basis for measurements of the real and imaginary parts of the coupling strength in grating translation experiments. In the case of local response, $p''=0$, we have $B^T=B_0^T=0$ and $A^T=A_0^T=\sin(p')/2$; here there is no effect of beam coupling. This matches with the above mentioned properties of the diffraction efficiency. For the nonlocal response, $p'=0$, we have $A^T=A_0^T=0$, $B^T=\tanh(p'')/2$, and $B_0^T=\sin(p')/2$. The coupling effects become important for $p' \gtrsim 2$, see Fig. 3.

C. Reflection geometry

In this case the input amplitudes of the writing beams are S_0 and R_d, see Fig. 1(d). The parameters $(\beta_d\beta_0)^{1/2}$, $(\beta_d/\beta_0)^{1/2}$, and ψ entering Eqs. (21) and (24) can be expressed by p'', the ratio p'/p'', and the input intensity ratio $\beta_{in}=|R_d|^2/|S_0|^2$. The corresponding relations, obtained by solving Eqs. (3) and (25), are

$$(\beta_d\beta_0)^{1/2} = \beta_{in}\, e^{p''},$$

$$(\beta_d/\beta_0)^{1/2} = (\beta_{in}\, e^{p''} + e^{-p''})/(1+\beta_{in}),$$

$$\psi = (p'/p'')\ln(\beta_d/\beta_0)^{1/2}. \qquad (31)$$

Since the influence of coupling effects on the readout properties was never analyzed in the R case, we consider this issue in some details.

1. Diffraction efficiency.

An explicit relation for η can be obtained by substituting Eqs. (31) into Eq. (21). The expression for η relevant to the uniform-grating model (Kogelnik theory), to be compared with, is $\eta=\tanh^2(m_{in}|p|/2)$, where the input light contrast $m_{in}=2\beta_{in}^{1/2}/(1+\beta_{in})$. A number of particular cases are of interest.

The limit of nonlinearly thin crystal, $|p|=|\gamma\,\kappa\,d| \lesssim 1$. The exact theory and the uniform-grating model give here the same result, $\eta \simeq (m_{in}|p|/2)^2 \lesssim 1$, and the coupling effects are negligible.

The case of local nonlinear response, $p''=0$. The energy exchange between the recording beams is absent here and the only nonlinear factor affecting diffraction is the modulation of the fringe positions. Using Eqs. (21) and (31) we obtain

$$\eta^{-1} = 1 + \frac{(\beta_{in}-1)^2}{4\beta_{in}\sin^2[p'(\beta_{in}-1)/2(\beta_{in}+1)]}. \qquad (32)$$

In the limit $\beta_{in} \to 1$ the numerator and denominator of this expression tend to zero. By resolving the $0/0$ indefiniteness we have $\eta=p'^2/(4+p'^2)$. Accordingly, the diffraction efficiency grows monotonously with increasing $|p'|$ and approaches unity.

The solid lines in Fig. 4(a) show the exact dependence $\eta(\beta_{in})$ in a logarithmic scale for $p''=0$ and three representative values of p'. The dotted lines show the dependences relevant to the uniform-grating model. All the curves are symmetric to the replacement β_{in} by β_{in}^{-1}. For $|p'|=1$ the influence of the coupling effects is small, but it is well pronounced for $p' \gtrsim 2$. The coupling effects always decrease the diffraction efficiency. By comparing Fig. 4(a) with Fig. 2(a), one can see not only similarities but also qualitative and quantitative differences.

The nonlocal response, $p'=0$. The phase ψ is zero in this case so that the fringe positions remain unchanged. At the same time, the light contrast and the grating amplitude are modulated across the crystal because of the energy exchange. The diffraction efficiency is given by the expression

$$\eta = \frac{m_{in}\sinh^2(p''/2)}{\cosh(p''+\ln \beta_{in}^{1/2})}. \qquad (33)$$

It is not an even function of p'' and $\ln \beta_{in}$. The limiting values of η for $p'' \gg 1$ and $p'' \ll 1$ are $(\beta_{in}+1)^{-1}$ and $\beta_{in}(\beta_{in}+1)^{-1}$, respectively. Figure 4(b) shows the function $\eta(\lg \beta_{in})$

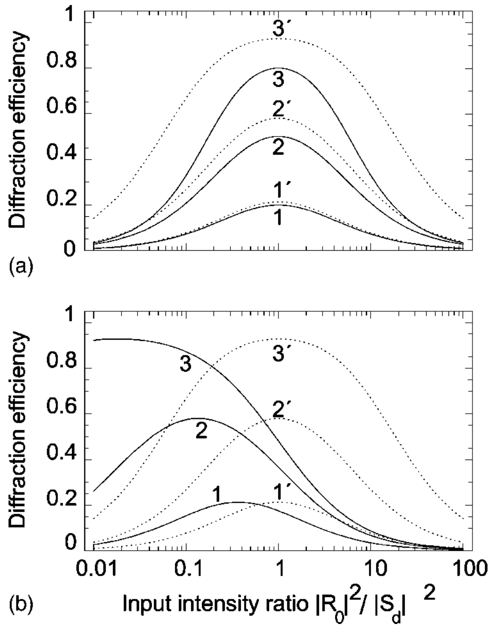

FIG. 4. Diffraction efficiency vs the input intensity ratio for the R geometry and the (a) and nonlocal (b) nonlocal nonlinear response. The dotted lines correspond to the uniform-grating model. The curves $1,1'$, $2,2'$, and $3,3'$ are plotted for the values of the coupling strength 1, 2, and 4, respectively.

for three representative values of p''. Its maximum is shifted to the left for $p''>0$ and to the right for $p''>0$; the larger is the value of the coupling strength, the stronger this shift. It is interesting that coupling does not affect the maximum value of $\eta(\beta_{in})$, compare the solid and dotted lines. Qualitatively, the solid curves in Figs. 2(b) and 4(b) look similar but there are quantitative differences. In particular, for the same coupling strength $|p''|$ the maximum achievable value of η is higher in the R geometry.

Equal input intensities, $\beta_{in}=1$. Here we have

$$\eta=\frac{1}{2}\frac{\cosh p''+(\cosh p'')^{-1}-2\cos\psi}{\cosh p''-\cos\psi}, \quad (34)$$

with $\psi=(p'/p'')\ln(\cosh p'')$. This expression is useful to analyze the features of the mixed nonlinear response. Figure 5

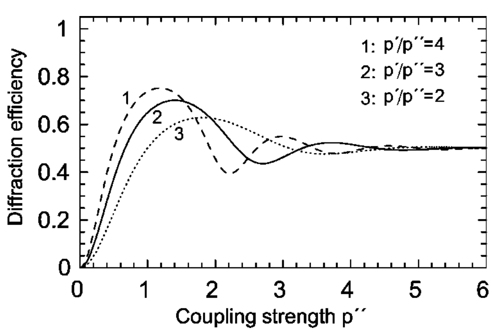

FIG. 5. Dependence $\eta(p'')$ for the input intensity ratio $\beta_{in}=1$ and different kinds of nonlinear response; the curves 1, 2, and 3 are plotted for $p'/p''=2,3$, and 4, respectively.

shows the dependence $\eta(p'')$ for two different values of the ratio p'/p''. This dependence is symmetrical, inversion of p'' does not affect η. For small values of the ratio p'/p'' the function $\eta(|p''|)$ increases monotonously from 0 to 1/2. With increasing $|p'/p''|$ this function becomes oscillating but its limiting value for $|p''|\to\infty$ remains equal 1/2.

2. Grating translation technique.

We restrict ourselves to the simplest case of equal input intensities, $\beta_{in}=|R_d|^2/|S_0|^2=1$. Using Eqs. (24) and (31) it is easy to obtain for the coefficients A^R, B^R entering Eq. (23):

$$A^R=\frac{1}{2}\frac{\tanh p''\sin\psi}{\cosh p''-\cos\psi}, \quad B^R=\frac{1}{2}\tanh p'', \quad (35)$$

where, as earlier, $\psi=(p'/p'')\ln[\cosh(p'')]$. These relations have to be compared with the prediction of the uniform-grating model,

$$A_0^R=\frac{p'\tanh(|p|/2)}{|p|\cosh(|p|/2)}, \quad B_0^R=\frac{p''\tanh(|p|/2)}{|p|\cosh(|p|/2)}. \quad (36)$$

For nonlinearly thin crystals, $|p|=|\kappa\gamma d|\lesssim 1$, Eqs. (35) and (36) give indeed the same result.

In the case of local response, $p''=0$, we obtain for the intensity changes using Eqs. (23) and (24):

$$\frac{\delta I_{S,R}}{I_{in}}=\pm\frac{2p'}{4+p'^2}\sin\varphi. \quad (37)$$

The oscillation amplitude $A^R(p')$ grows first as $p'/2$, experiences a maximum at $p'=2$ (where $\delta|I_{S,R}|/I_{in}=1/2$, and decreases then with increasing p', see Fig. 6(a). An agreement with the uniform-grating model takes place for $|p'|\lesssim 2$.

For the nonlocal response, $p'=0$, we have

$$\frac{\delta I_{S,R}}{I_{in}}=\pm\frac{\tanh p''}{2}(1-\cos\varphi). \quad (38)$$

The oscillation amplitude $B^R(p'')$ approaches monotonously the values $\pm 1/2$ for $p''\to\pm\infty$. The uniform-grating model is well applicable for $|p''|\lesssim 2$, see Fig. 6(b).

V. DISCUSSION

The most general outcome of this study is in reducing the problem of readout of dynamic index gratings to the recording problem regardless of particular properties of the nonlinear medium. The explicit relations found in Sec. III express the desirable readout characteristics directly through a restricted set of data on the input and output recording amplitudes.

A different approach has already allowed to investigate the dynamics of the feedback-controlled beam coupling and to obtain a number of new particular results for the reflection coupling geometry, see Sec. IV. The use of this approach is not, however, restricted to these examples. It can, in particular, be applied to the transient processes of grating formation; recording characteristics of these processes admit often a complete analytical study [2].

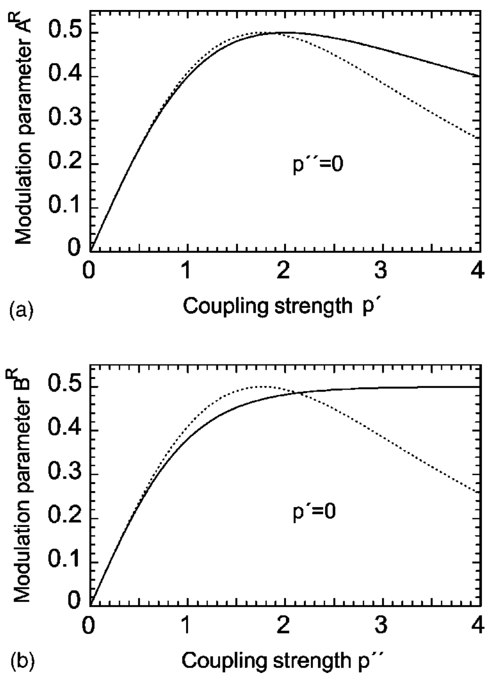

FIG. 6. Dependence of the modulation amplitudes A^R (a) B^R (b) on the coupling strength for the local and nonlocal response, respectively, in the R geometry; the dotted lines are plotted for the uniform-grating model.

The general relations between the readout and recording characteristics can be applied to the problem of modeling of the material relations for the grating amplitude. These relations are often the bottleneck of the photorefractive studies because of complexity of the charge separation processes. The supplementary information on the recording process, gained by measuring the readout characteristics, allows one to judge about validity of models of the nonlinear response.

Some restrictions on our approach are worthy of discussion. One of them is the assumption of scalar beam coupling, i.e., the absence of polarization coupling. This assumption is valid for anisotropic photorefractive crystals such as $LiNbO_3$, $BaTiO_3$, SBN, but is not correct for cubic materials, such as the sillenite crystals [Bi_{12}(Si,Ti,Ge)O_{20}], and semiconductors GaAs, CdTe, see Ref. [19] and references therein. Moreover, specific readout properties of vectorial beam coupling in cubic materials have found recently an important application for detection of weak signals [20–22]. Most probably, our method can be generalized to include the effects of polarization coupling.

One more restriction concerns with the neglect of light absorption effects, especially the absorption gratings. Fortunately, such effects are often relatively small because the values of the $\kappa\gamma$ product, characterizing the rate of the spatial changes of the light amplitudes, exceed considerably the values of the light absorption coefficient. The influence of the absorption gratings can be taken into account within the uniform-grating model, i.e., for weak or modest coupling strength [17].

VI. CONCLUSIONS

A different method for description of the readout processes for dynamic photorefractive index gratings is developed. Within this method, the readout characteristics are expressed explicitly through the input and output recording amplitudes by making use of the symmetry properties of the coupled-wave equations and with no use of particular material relations for the nonlinear response. The approach changes the status of the readout problem. It is proven to be applicable to a broad range of particular photorefractive effects and techniques.

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